NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

APPLICATION OF VARIANCE REDUCTION
TECHNIQUES TO THE COMPUTER SIMULATION
OF AN OUTPATIENT CLINIC

by

Karl Hugo Eulenstein

September 1974

Thesis Advisor: R. W. Butterworth

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Application of Variance Reduction Techniques

to the Computer Simulation of an Outpatient Clinic

by

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requirements for the degree of

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ABSTRACT

A statistical analysis of the Silas B. Hays Hospital, North Fort Ord Family Practice Clinic was conducted over a period of five months of operation. A computer simulation model of the clinic was constructed. The response of the model's statistical measure of effectiveness to increased input arrival rates and the addition of one physicians' assistant to the staff was measured in a sensitivity analysis. Control variate and antithetic variance reduction techniques were applied to the model and compared with the ordinary Monte Carlo method for their effectiveness in the reduction of variance of the output statistic.

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I. INTRODUCTION

A. BACKGROUND

A recent development in the practice of medicine has been the renewed emphasis on a continuous and personal relationship between patient and doctor. Manifestations of this trend are the many physicians trained in the new family practice specialty, serving in the military as well as the civilian sector. The principal underlying theme of this new specialty has been to provide continuous care to patients within a family doctor framework. In fact, the Family Practice Physician is a recognized specialist who has been trained to provide total family care usually encompassing acute minor illnesses, pediatrics, obstetrics, minor surgery, gynecology, internal medicine and family counseling. Although no definitive statistics are available, it has been estimated that approximately 80% of an outpatient's medical demands can be met by a family practitioner.

The North Fort Ord Family Practice Clinic, hereafter referred to as FPC, is the most recent clinic of its type to be established at Fort Ord, California. Its operation is very similar to that of the family practice clinic located in Silas B. Hayes Hospital, also at Fort Ord [Dilley and Larkins 1973]. Its primary resources are four doctors with an attendant staff of one nurse clinician, several nurses, aides and receptionists. A patient population of about 4000 people, or 1000 families, was being served by the FPC during

the period of the study, October 1973 to April 1974. Included in this population were active duty, dependent and retired personnel of all ages and from all branches of the armed forces.

B. PROBLEM STATEMENT

The staff of the FPC routinely had collected statistical information about its operation since opening in August 1973. In order to become a useful management tool, these statistics had to be analyzed and summarized. Thus the first portion of the problem was to make a comprehensive statistical analysis of the FPC operation over a period of five months.

An important problem for the FPC administrators was determination of the largest population size for which a four doctor clinic could provide quality care. It was assumed that quality care was being provided at the time of the commencement of this study and that this quality would not be affected as long as the average amount of time a physician spent caring for a patient did not decrease and the waiting lines did not grow at an ever-increasing rate. Since arbitrary changes in population size were totally impractical, the second portion of the problem was to determine, through computer simulation, an estimate of the projected effects of increasing the number of patients assigned to each doctor.

The military physicians' assistant (PA) is a skilled paraprofessional health care provider whose responsibility level lies somewhere between a nurse clinician and a physician. Although no PA's had been assigned to the FPC, the

staff expressed a need to know what effect the introduction of a PA would have on the clinic's patient population capacity. Therefore, the third portion of the problem was to estimate the probable impact of a PA.

II. STATISTICAL ANALYSIS

A. DATA COLLECTION

Although statistics for the FPC from August 1973, the clinic's beginning month, through March 1974 were available for analysis, only October, November, January, February and March were considered. August and September were not selected because the clinic was too young to have reached a tempo of operation approaching steady state. December was omitted due to the unusually low number of patient encounters during the Christmas holiday season.

The data which were available consisted of arrival logs, encounter forms and the appointment calendar. Arrival logs were kept by the receptionists who recorded each arriving patient's identification, arrival time and the nature of the complaint. Encounter forms were initiated by the receptionist for each patient and completed by the staff members of the clinic with whom the patient interacted during his visit. The most recent version of the encounter form is reproduced in Figure 1. A blank page of the appointment calendar is reproduced in Figure 2.

The encounter forms and appointment calendar pages were collected and filed by the statistical staff of the clinic. A cross check of patient totals derived from the encounter forms with the totals derived from the appointment calendar revealed a disagreement of less than 0.3%. Thus ample and complete data were available for the analysis. All of the

HEALTH CARE STUDIES UNIT
NORTH FAMILY PRACTICE CLINIC

1. Date:________________

2. Patient's Name:______________________________

3. Sponsor's SSAN (with patient's prefix):

☐☐ ☐☐☐☐☐ ☐☐ ☐☐

4. Appointment Status:

25 (20)Emergency (21)Appointment (22)Walk-in

6. Health Care Provider(name or #):______________________________

7. Patient's Complaint:______________________________

8. PROVIDER TIME ON PROBLEM

Most Time		Second Most
(160)	Acute injury	(180)
(161)	Acute injury followup	(181)
(162)	Acute(temporary) problem	(182)
(163)	Acute(temporary) problem f/u	(183)
(164)	Chronic problem, routine	(184)
(165)	Chronic problem, flare-up	(185)
(166)	Prenatal & postnatal care	(186)
(167)	Pa[illegible] exam, well baby, [illegible] lab, or prev. ed.	(187)
(168)	[illegible] Hx and PE finished	(188)
(169)	Fam[illegible] ng/Contraception	(189)
(170)	Counseling [illegible]	(190)
(171)	Immunization	(191)
(172)	Administrative	(192)
(173)	Other	(193)

(22_)PHARMACY: # of Rx______

(231)EKG

9. LENGTH OF THIS VISIT

(210) 0 - 5 min.
(211) 5 - 20 min.
(212) 20 - 40 min.
(213) Over 40 min.

10. X-RAY

(240)Chest-P.A.
(241)Chest-P.A. & Lat.

Other______________________

____

Other Nursing Care______________________

____

____

11. LAB

(350)SMA-12, fasting	(360)CBC & Diff.
(351)SMA-12, non-fasting	(361)CBC
(352)Chol & Trigly	(362)Hct.
(353)Electrolytes	(363)Sickle Cell
(502)K+	(370)G6PD
(521)Na+	
(354)Glucose, fasting	(364)Urinalysis
(355)Glucose,___hr pp only	(36[illegible])Clean catch UA
(356)Glucose 1 or 2 hr [illegible] sugar meal	(3[illegible])Urine culture
	([illegible])Throat culture
(357)[illegible]	
(358)Mononucl. Screen	(368)GC screen
(359)[illegible]bella [illegible]	(36[illegible])PAP
(374) [illegible] & T_4	(504)Pregnancy test

Other Lab______________________________

____

12. IMMUNIZATIONS

(610)OPV	(615)MMR
(611)DPT	(616)Measles & Rubella
(612)DT	(617)Mumps
(613)T. Tox	(618)Flu
(614)Tine	(619)Smallpox

Other______________________

13. REFER TO

(700)Dental	(709)Ophthalmology
(701)Dermatology	(710)Optometry
(702)ENT	(711)Orthopedics
(703)Internal Med	(712)Pediatrics
(704)Ment Hyg/Soc Wk	(71[illegible])Preventive Med.
(716)Nuclear Med.	(713)Psychiatry
(705)Neurology	(714)Surgery
(706)OB-GYN	(715)Urology
(707)OT/PT	

Other____________

NEXT APPOINTMENT

15 min.________________

30 min.________________

45 min.________________

60 min.________________

30 min with Nurse Clin________

P.E. with Nurse Clin________

Other________________

MF 557 Rev 14 Dec 73

Figure 1

SILAS B. HAYS ARMY HOSPITAL
FORT ORD, CALIFORNIA 93941
MF 522, 13 JUL 72

DAY | MO | DOCTOR

RECORDS	IN PATIENT	ARMY	NAVY - MARINE CORPS	AIR FORCE	USCG	USPHS	PATIENT	NUMBER	AGE	UNIT / PHONE	
										*	08 00
											10
											30
										*	40
											09 00
											10
											30
											40
											10 00
										*	10
										*	30
											40
											11 00
											10
											30
										*	40
											13 30
										*	40
											14 00
											10
											30
											40
										*	15 00
										*	10
											30
											40
											16 00
										*	10

Figure 2

data taken pertained only to the principal resources of the clinic -- the four physicians -- and did not include night-time, off-duty or weekend encounters.

B. ENCOUNTER DATA

The patient encounter data taken from 5455 encounter forms, averaged over five months, for all four doctors, are summarized in Table I.

In an attempt to make certain that the population of patients allocated to each doctor mirrored the age, sex and status distribution of the total patient population, each new family was assigned a doctor on a random basis. Nevertheless, the doctors' panels were not identical in size. The five month average number of patient encounters per month per doctor, as shown in Table II, was found to be proportional to the total number of patients on the doctor's panel.

C. APPOINTMENT CALENDAR

During the past ten years, the most productive analyses of outpatient clinic utilization and efficiency have concerned themselves with the clinics' appointment calendars. An incontrovertible conclusion of this work has been that a poorly designed appointment system inevitably results in reduced physician utilization and increased patient waiting times which contribute to patient dissatisfaction [Stimson and Stimson 1972]. The structure of the appointment calendar directly influences the number of patients who are seen

TABLE I

ENCOUNTER DATA SUMMARY

ENCOUNTERS PER MONTH		1089
ENCOUNTERS PER WORKDAY		55
DAILY DISTRIBUTION (%)	MONDAY	23
	TUESDAY	18
	WEDNESDAY	20
	THURSDAY	20
	FRIDAY	19
TIME OF DAY (%)	0800-1200	63
	1330-1630	37
SEX OF PATIENT (%)	MALE	38
	FEMALE	62
AGE OF PATIENT (%)	0-12	26
	13-18	8
	19-35	34
	36-50	20
	OVER 50	12
STATUS (%)	SPONSOR	21
	SPOUSE	44
	DEPENDENT	35
APPOINTMENT STATUS (%)	SCHEDULED APPOINTMENT	92
	WALK-IN	8
EMERGENCIES (%)		1

by a doctor on a given day, the amount of time the doctor devotes to each patient, the waiting times of the patients and the idle time of the doctor.

TABLE II

MONTHLY DOCTOR ACTIVITY

DOCTOR	ENCOUNTERS PER MONTH	%
#1	313	29
#2	276	25
#3	279	26
#4	225	21

In addition to its use for the routine scheduling of patients, the calendar at the FPC was used to record the occurrence of doctor-patient encounters, canceled appointments, patients who failed to appear for appointments and the allocation of the doctors' time for other than routine medical practice. Consequently, this calendar information became the prime statistical source in the effort to determine the distribution of time allotted by the principal resource -- the doctors -- to various activities throughout a normal working day.

A page of the appointment calendar is shown in Figure 2. The fundamental time unit was 15 minutes, with appointments scheduled for 15, 30, 45 or 60 minutes. The duration of an appointment was determined by the physician according to the nature of the patient's problem. Although time was allotted

in 15 minute increments, the appointments were scheduled to begin on the hour, and at 10, 30 and 40 minutes after the hour. This scheduling method permitted the simultaneous use of two examining rooms per doctor to undress and prepare patients before they were actually seen by the doctor. A maximum of 28 time periods, each of 15 minutes, were available every day. Appointment times shown with an asterisk were withheld for acute illnesses that required a doctor's attention within less than 24 hours. Thirty-two percent (nine periods) of the total number of 15 minute time periods in each day were reserved in this manner for semi-emergencies.

The distribution of time, in minutes, which all four doctors devoted to various activities is given in Table III. The DOCTOR-PATIENT column was the time spent caring for patients. LATE CANCEL NO-SHOW was unused time due to appointments that were canceled too late to be rescheduled for another patient and appointments for which patients simply failed to appear. NOT SCHEDULED was time not scheduled either because patients were not able to accept appointments at available times on the calendar or because the number of semi-emergencies was insufficient to fill up the nine daily time periods allotted for them. OTHER time was allocated for annual leave, medical research, conferences and other bona fide activities.

The distribution of the length of scheduled appointments for each doctor was also derived from the appointment calendar. Table IV summarizes this data, averaged over five

TABLE III

MONTHLY TIME DISTRIBUTION

MONTH	DOCTOR IS "IN"			DOCTOR "OUT"
	DOCTOR-PATIENT	LATE CANCEL NO-SHOW	NOT SCHEDULED	OTHER
OCT	19605 min.	2385	3045	10245
NOV	19140	2910	3045	10185
JAN	20775	2190	3180	5775
FEB	17640	2355	2175	9750
MAR	22845	2250	3240	6945
TOTAL	100005	12090	14685	42900
MONTHLY AVERAGE	20001	2418	2937	8580
% OF TOTAL TIME	58.9	7.1	8.7	25.3
% OF "IN" TIME	78.9	9.5	11.6	-----

TABLE IV

DISTRIBUTION OF APPOINTMENTS

DOCTOR	MINUTES 15	30	45	60	AVERAGE TIME
#1	85%	14%	1%	0%	17.4 min.
#2	78	16	4	2	19.5
#3	76	17	5	2	20.0
#4	86	11	2	1	17.7
OVERALL	81	15	3	1	18.6

months. Entries are percentages of the total number of each doctor's appointments. AVERAGE TIME is the mean time per scheduled appointment in minutes.

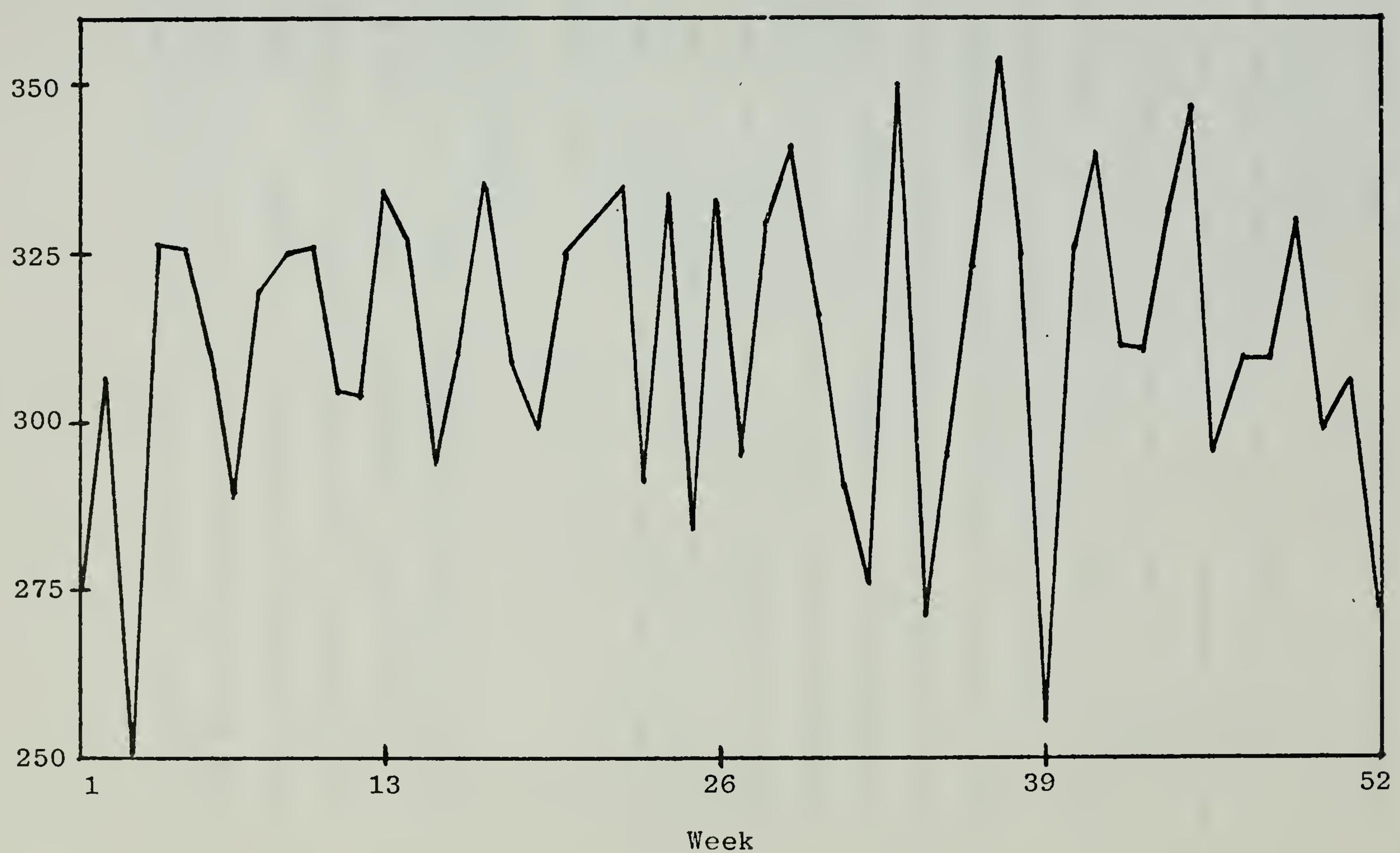
III. SIMULATION MODEL

A. DESCRIPTION

The computer simulation model of the FPC was constructed using the General Purpose Simulation System (GPSS) language. A listing of the program appears in Appendix A. The data used to build the model were derived primarily from the used FPC appointment calendars. A weekly demand rate for out-patient health care was estimated from a five percent random sample of a one million member civilian prepaid health plan during 1971. In queueing theory terminology, the model consisted of four parallel M/G/1 queues for the four doctors plus an additional M/G/1 queue for the physicians' assistant. The statistical measure of effectiveness of interest was the number of patients remaining in each queue at the end of each week, and their average over the 52 weeks of a year.

The civilian health plan data indicated that approximately four appointment requests per patient per year could be expected on the average. However, the intensity of these demands was not constant over time. Figure 3 shows the non-homogeneous arrival pattern over one year which was incorporated in the model after adjustments for the FPC population size. Weekly arrivals were assumed to be distributed Poisson with a mean fluctuating as in Figure 3. The arrivals were distributed among the four doctors in proportion to each doctor's alloted portion of the patient population. Thus the four doctors received 31%, 24%, 24% and 21% of all appointment requests respectively.

Figure 3. Weekly Patient Arrivals



Service time was interpreted to be the amount of appointment calendar time allocated to an appointment. The empirical distribution of these times was taken directly from the appointment calendar data.

The appointment calendar contained 28 fifteen minute periods each day for five days each week. Consequently, the simulation run length was 2100 minutes per week or 109200 minutes each year. The arrival rate was adjusted at the beginning of each week to conform to the rates depicted in Figure 3. The number of patients remaining in each doctor's queue at the end of each week were printed in matrix form at the end of each yearly run. In addition, the percentage of calendar time not scheduled, time allotted for appointments which were kept and idle time due to late cancellations and no-shows as well as calendar utilization were also printed in matrix form. Appendix B contains an annotated example of the simulation output.

B. SENSITIVITY ANALYSIS

Initial runs were conducted without the physicians' assistant in order to validate the computer program and to conduct a sensitivity analysis under current operating consitions. Table V compares the calendar utilization of the simulation model with the utilization derived from the data. All of the percentages of error were less than or equal to 1.1%.

TABLE V

UTILIZATION COMPARISON

QUEUE	MODEL	DATA	ERROR	% ERROR
Doctor #1	92%	91%	+1	1.1%
Doctor #2	87	87	0	0.0
Doctor #3	89	90	-1	1.1
Doctor #4	87	87	0	0.0
TOTAL SYSTEM	89	89	0	0.0

The weekly queue lengths displayed in the output matrix were examined to ascertain the effects of the non-homogeneous patient arrival rate. A typical output matrix is shown in Appendix B. Queue lengths tended to increase to a relative maximum during the first twenty weeks of simulation -- the influenza season -- and decreased to a relative minimum during the summer months. A gradual increase in queue length began again as the winter months approached.

In order to estimate the maximum patient capacity of the system, the arrival rate was accelerated until the individual calendars of the doctors, and eventually the total system, began to form infinitely growing queues. Table VI depicts calendar utilization as a function of increased arrival rates from a simulated ten year sample. As the individual utilizations increased beyond 100%, the corresponding calendar queues began to grow at a monotonic, non-decreasing rate. These overload points are shown in the last column of Table VI.

TABLE VI

CALENDAR UTILIZATION AT INCREASED ARRIVAL RATES

QUEUE	LOAD (% OF NORMAL)					OVERLOAD POINT
	100%	104%	110%	114%	118%	(% OF NORMAL LOAD)
#1	92%	94%	100%	100%	100%	110%
#2	87	90	95	97	100	118
#3	89	92	97	100	100	114
#4	87	89	95	98	100	118
TOTAL SYSTEM	89	91	97	98	100	116

C. PHYSICIANS' ASSISTANT

The physicians' assistant was introduced into the simulation model in order to estimate his potential impact on the patient capacity. It was assumed that the PA's calendar would operate at a level of utilization slightly less than that of a doctor. Several trial runs using a mean service time of 30 minutes per PA appointment showed that assigning 14% of each doctor's patient arrivals to the PA resulted in a PA utilization which was one or two percent lower than the total system utilization.

Addition of the physicians' assistant had a dampening effect on the weekly and seasonal fluctuations in queue length. In addition, the system's total patient capacity was increased substantially. Arrival rates were accelerated to estimate, as before, when the queues would begin to grow monotonically. The results are given in Table VII. The

point at which the total system queue began to grow at an ever-increasing rate occurred at an arrival rate that was 23% greater than the arrival rate for the same point of the system without the PA.

TABLE VII

CALENDAR UTILIZATION AT INCREASED ARRIVAL RATES WITH PA

QUEUE	LOAD (% OF NORMAL)					OVERLOAD POINT (% OF NORMAL LOAD)
	100%	121%	128%	138%	141%	
#1	80%	97%	100%	100%	100%	128%
#2	75	90	96	100	100	132
#3	78	94	98	100	100	130
#4	76	92	98	100	100	130
PA	72	89	93	98	100	139
TOTAL SYSTEM	76	92	97	99	100	139

IV. VARIANCE REDUCTION

As the utilization of the physicians' calendar was increased, the sample variance of the output statistic also became larger. Table VIII shows how the variance of the queue length estimator increased as the arrival stream was accelerated beyond the current operating level of the clinic. In an attempt to reduce this variance, two variance reduction schemes were applied. The first of these was the method of control variates; the second, antithetic variates.

TABLE VIII

EXPECTED QUEUE LENGTH VARIANCE

TEN 1-YEAR ITERATIONS

PATIENT ARRIVALS	DOCTOR 1	DOCTOR 2	DOCTOR 3	DOCTOR 4	PA	SYSTEM	SYSTEM UTIL. (%)
BASE	0.8	0.3	0.9	0.0	0.4	1.0	76.3
+21%	152.9	15.3	115.4	54.5	12.3	250.4	92.1
+28%	1741.1	104.4	183.0	724.5	101.1	2045.4	96.9

A. METHOD OF CONTROL VARIATES

The method of control variates involved the use of a second model whose simulation could be positively correlated with the original model and for which the mean value of the estimator for mean queue size could be computed analytically [Gaver and Thompson 1973]. A simple theoretical model which

closely approximated the results of the original model had to be constructed. The equation for the queue size estimator was:

$$Q = E(A) + \overline{Q} - \overline{A}$$

where

$E(A)$ = known expected queue length for the control model.

$\overline{Q}$ = mean queue length empirically found with the simulation model.

$\overline{A}$ = mean queue length empirically found with the control model.

The crucial point of this method was that identical sequences of pseudo-random numbers had to be used in the simulations to compute $\overline{Q}$ and $\overline{A}$ in order to ensure a positive correlation between these two statistics. Q was an unbiased estimator since $E(A) = E(\overline{A})$, and its variance was:

$$V(Q) = V(\overline{Q} - \overline{A}) = V(\overline{Q}) + V(\overline{A}) -2\ \mathrm{Cov}(\overline{Q},\overline{A})$$

A positive correlation between $\overline{Q}$ and $\overline{A}$ implies a positive covariance. This in turn should decrease the variance of Q to a value less than the variance which could be realized, namely $V(\overline{Q})$ using the ordinary Monte Carlo method, provided that $2\ \mathrm{Cov}(\overline{Q},\overline{A})$ was greater than $V(\overline{A})$.

The control model used to approximate the simulation model of the clinic was a system of five parallel M/M/1 queues operating at the same average activity level as the original model of the clinic. None of the refinements such as unused

time due to no-shows, late cancellations nor a time dependent arrival stream were included. The system's level of utilization chosen for this experiment was 92% which implied a patient arrival stream 21% above normal. Twenty Monte Carlo iterations of the original model were compared with ten iterations of the control variate method involving approximately the same amount of effort and total computer time. The results are shown in Table IX. Clearly, the desired variance reduction was not achieved. Reasons for this outcome are addressed in the section entitled RESULTS OF VARIANCE REDUCTION.

TABLE IX

VARIANCE OF QUEUE LENGTH ESTIMATOR

METHOD OF CONTROL VARIATES

QUEUE	MONTE CARLO 20 ITERATIONS	CONTROL VARIATE 10 ITERATIONS
Doctor 1	119.6	749.2
Doctor 2	15.5	16.5
Doctor 3	89.4	174.9
Doctor 4	33.3	80.0
PA	13.2	18.1
TOTAL SYSTEM	216.0	642.8

B. METHOD OF ANTITHETIC VARIATES

The method of antithetic variates involved the use of a second queue length estimator calculated from antithetic versions of the original random number stream. Its expected value was the same as that of the original estimator and its correlation with the original estimator should be negative [Hammersley and Handscomb 1964]. Negative correlation was achieved by using two sets of antithetically generated random numbers for two companion simulation runs respectively. That is, a set of random numbers R, distributed uniformly on the interval [0,1], was used for ten one-year iterations of the simulation model. Then a second set of ten iterations of the same model was run using the set of random numbers 1-R.

The antithetic estimators were:

$$Q = \frac{\overline{Q} + \overline{A}}{2}$$

where

$\overline{Q}$ = sample average queue length using the set R.

$\overline{A}$ = sample average queue length using the set 1-R.

The estimator is unbiased since:

$$E(Q) = \tfrac{1}{2}[E(\overline{Q}) + E(\overline{A})] = \tfrac{1}{2}[E(Q) + E(Q)]$$

Its variance is reduced since:

$$V(Q) = V[\tfrac{1}{2}(\overline{Q}+\overline{A})] = \tfrac{1}{4}[V(\overline{Q}) + V(\overline{A}) + 2\ \mathrm{Cov}(\overline{Q},\overline{A})]$$

and $\mathrm{Cov}(\overline{Q},\overline{A}) \leq 0$.

If the relationship between the random number R and $\overline{Q}$ as well as between 1-R and $\overline{A}$ was monotonic, then the negative correlation between 1-R and R would result in a negative covariance between $\overline{Q}$ and $\overline{A}$. This, in turn, would result in a reduction of the variance of Q to a value less than that which could be expected using the ordinary Monte Carlo method.

Two antithetically generated companion runs of ten one-year iterations, in which antithetics were applied to the generation of inter-arrival times as well as service times, were compared with 20 iterations of the Monte Carlo method, involving approximately the same amount of effort and computer time. System level of utilization was 92%. The resultant variance reductions are shown in Table X.

TABLE X

VARIANCE OF QUEUE LENGTH ESTIMATOR

METHOD OF ANTITHETIC VARIATES

QUEUE	MONTE CARLO 20 ITERATIONS	ANTITHETIC 10 ITERATIONS	% REDUCTION
Doctor 1	119.6	77.0	36
Doctor 2	15.5	7.7	50
Doctor 3	89.4	50.4	44
Doctor 4	33.3	21.7	35
PA	13.2	3.1	77
TOTAL SYSTEM	216.0	86.7	60

A second set of antithetic runs of ten iterations in which antithetics were applied only to the generation of inter-arrival times resulted in the variance reductions shown in Table XI.

TABLE XI

ESTIMATOR VARIANCE

ANTITHETIC ARRIVALS

QUEUE	MONTE CARLO 20 ITERATIONS	ANTITHETIC 10 ITERATIONS	% REDUCTION
Doctor 1	119.6	88.5	26
Doctor 2	15.5	7.7	50
Doctor 3	89.4	28.7	68
Doctor 4	33.3	14.3	57
PA	13.2	4.8	64
TOTAL SYSTEM	216.0	115.8	46

The results of a final set of runs in which antithetics were applied only to the generation of appointment times are shown in Table XII.

TABLE XII

ESTIMATOR VARIANCE

ANTITHETIC APPOINTMENT TIMES

QUEUE	MONTE CARLO 20 ITERATIONS	ANTITHETIC 10 ITERATIONS	% REDUCTION
Doctor 1	119.6	20.4	83
Doctor 2	15.5	10.6	32
Doctor 3	89.4	37.6	58
Doctor 4	33.3	16.8	50
PA	13.2	1.6	88
TOTAL SYSTEM	216.0	63.9	70

V. SUMMARY OF RESULTS

A. RESULTS OF STATISTICAL ANALYSIS

Assuming that the doctor "OUT" time was fixed at a monthly average of 8580 minutes, as shown in Table III, the results of the statistical analysis indicated that a limited expansion of the clinic's patient capacity was possible. Regardless of the configuration of the appointment calendar, the late cancel/no-show rate of 9.5% of the doctors' "IN" time would be difficult to change. However, the 11.6% (2937 minutes per month) of unscheduled time was available for additional appointments. Dividing 2937 minutes by the overall average service time of 18.6 minutes per appointment yielded a maximum additional number of 158 appointments per month. This represented a theoretical upper bound equal to 115% of current operating conditions. Operating the clinic at this level would be equivalent to operating at a calendar utilization equal to 100%.

B. RESULTS OF SENSITIVITY ANALYSIS

At a simulated patient load between 114% and 118% of the clinic's current operating point(without the PA), all of the queues began to grow at ever-increasing rates, all unscheduled time became zero and the utilization of all the queues had reached 100%. This result was in agreement with the results of the statistical analysis. However, this theoretical upper bound was not feasible. The output matrix

which showed the weekly queue length fluctuations over the entire year, indicated that excessive queue lengths during the early months of the year were realized well before 100% system utilization occurred.

Discussions with the FPC staff had led to the assumption that a queue length greater than about 500 patients for the system, or 125 per doctor, would result in excessive waiting times for the patients, implying a decrease in the quality of care as originally defined in the problem statement. Acceleration of the simulation model's patient arrival rate showed that peak system queue lengths during May began to exceed 500 patients when the arrival rate was 10% higher than normal, at a system utilization of 97%. Table XIII shows the weekly average queue lengths of the total clinic (without PA) for 52 weeks measured over ten one-year samples at an arrival rate 10% above normal. During week 20, the average queue length peaked at 501.4.

The second series of simulation runs, which included the PA, indicated that utilization of all the queues had reached 100% between 38% and 41% above normal capacity. As in the case without the PA, the weekly fluctuations of queue length as depicted in the output matrix led to a reduced maximum patient capacity that was 29% above normal. Table XIV shows the weekly average queue lengths of the total clinic for 52 weeks measured over ten one-year samples at an arrival rate 29% above normal and a total system utilization of 96%. The maximum acceptable number of patients in the calendar queue

TABLE XIII

AVERAGE WEEKLY QUEUE LENGTH WITHOUT PA

Week	Queue Length	Week	Queue Length
1	13.5	27	197.8
2	75.7	28	188.8
3	61.4	29	180.1
4	109.8	30	163.4
5	149.9	31	154.4
6	190.3	32	142.1
7	129.2	33	134.5
8	176.6	34	122.0
9	219.2	35	119.5
10	261.3	36	74.4
11	307.0	37	80.4
12	355.6	38	76.9
13	404.3	39	74.8
14	377.5	40	116.8
15	356.1	41	101.7
16	382.7	42	92.7
17	441.5	43	78.2
18	410.5	44	111.2
19	460.8	45	165.6
20	501.4	46	201.7
21	475.3	47	133.4
22	392.0	48	113.3
23	369.4	49	163.3
24	344.3	50	203.4
25	317.3	51	169.9
26	288.7	52	111.7

TABLE XIV

AVERAGE WEEKLY QUEUE LENGTH WITH PA

Week	Queue Length	Week	Queue Length
1	5.0	27	260.4
2	84.4	28	248.8
3	50.2	29	234.4
4	131.0	30	221.4
5	187.2	31	215.2
6	247.6	32	211.6
7	141.2	33	196.2
8	207.6	34	171.0
9	285.2	35	152.0
10	366.8	36	99.4
11	403.2	37	99.4
12	468.6	38	95.2
13	536.8	39	109.2
14	489.2	40	168.0
15	456.4	41	143.2
16	525.2	42	131.6
17	590.0	43	129.6
18	543.4	44	188.0
19	592.8	45	269.6
20	645.8	46	349.4
21	588.2	47	238.6
22	497.0	48	202.4
23	464.0	49	275.0
24	439.0	50	365.8
25	404.2	51	315.6
26	376.8	52	221.8

had been increased to 625 to account for the addition of the PA. The system average queue length reached a maximum of 645.8 during the twentieth week.

Thus, the simulation results revealed a phenomenon that could not have been quantified analytically; namely, that increasing the patient capacity was limited by the time dependent fluctuations of queue length.

C. RESULTS OF VARIANCE REDUCTION TECHNIQUES

1. Control Variates

Variance reduction, using the method of control variates, was not realized. The most probable cause of this method's vailure was a lack of correlation between the main simulation model and the control model. To check correlation between the two models, the weekly queue lengths of the two models were compared over the period of one year of simulation. The results of this comparison are shown in Table XV. Only the total system queues are shown. However, doctor queue length differences between the two models were of approximately the same magnitude as total system queue length differences.

The sample correlation coefficient (r_{12}) was calculated using the following formulas:

$$r_{12} = \frac{s_{12}}{s_1 s_2}$$

$$s_{12} = \frac{1}{51} \sum_{k=1}^{52} (x_{1k} - \bar{x}_1)(x_{2k} - \bar{x}_2)$$

TABLE XV

WEEKLY SYSTEM QUEUE LENGTH

Week	Main Model (1)	Control Model (2)	Difference (1 - 2)	Week	Main Model (1)	Control Model (2)	Difference (1 - 2)
1	15	135	-120	27	5	128	-123
2	52	124	- 72	28	11	107	- 96
3	35	92	- 57	29	16	110	- 94
4	76	96	- 20	30	20	145	-125
5	90	66	24	31	22	145	-123
6	94	55	39	32	13	190	-177
7	36	51	- 15	33	33	162	-129
8	53	56	- 3	34	12	158	-146
9	85	38	47	35	19	124	-105
10	96	77	19	36	2	151	-149
11	136	67	69	37	15	161	-146
12	159	98	61	38	26	192	-166
13	185	86	99	39	25	181	-156
14	159	69	90	40	41	164	-123
15	134	63	71	41	24	146	-122
16	155	96	59	42	24	143	-119
17	207	86	121	43	3	139	-136
18	181	68	113	44	74	136	- 62
19	226	89	137	45	99	126	- 27
20	268	102	166	46	110	72	38
21	198	115	83	47	44	61	- 17
22	81	133	- 52	48	40	44	- 4
23	66	105	- 39	49	55	42	13
24	35	117	- 82	50	81	43	38
25	28	150	-122	51	34	29	5
26	31	135	-104	52	24	43	- 19

$$s_i = \left[\frac{1}{51} \sum_{k=1}^{52} (x_{ik} - \overline{x}_i)^2\right]^{\frac{1}{2}}$$

$$\overline{x}_i = \frac{1}{52} \sum_{k=1}^{52} x_{ik}$$

$$i = \begin{cases} 1 & \text{MAIN MODEL} \\ 2 & \text{CONTROL MODEL} \end{cases}$$

x_{ik} = queue length of column i, week k.

For the two model runs of 52 weeks depicted in Table XV, the sample correlation coefficient was -0.38, indicating that the correlation between the two models was not strong.

The lack of correlation was attributable, in part, to the fact that the patient arrival stream of the control model was not time dependent, whereas it was in the main model. The weekly queue length of the main model rose to a peak of 268 during week 20, while the queue length peak of 192 of the control model occurred during week 38. A comparison check of nine additional 52-week runs showed that the relative positions of the queue length peaks of the two models did not change significantly. Therefore, making the control model patient arrival stream non-homogeneous with respect to time was investigated.

The control model required 220 K bytes of computer core and about 20 minutes of total computer time per ten-year run. Although it was much more complex, the original model also required 220 K bytes and approximately 55 minutes by comparison. Introducing the time dependence into the

control model involved a large increase in its complexity and lengthening its run time to about 50 minutes. Moreover, a simple analytical solution for the expected queue length of the control model was no longer obvious.

Finally, to ensure that the patient arrivals of each model were in complete synchronization, the two models had to be combined into one computer program in which each patient arrival was split into two arrivals, one per model, having identical arrival times.

The sum total of these changes resulted in a very complicated program requiring approximately 440 K bytes of core and a run time of about 100 minutes, without a known expected value for the control model output statistic. Hence, the possibly variance reduction advantages were completely overshadowed by the new program's enormity, and this approach was abandoned.

2. <u>Antithetic Variates</u>

The success of the method of antithetic variates in reducing the variance of the queue length statistic was due primarily to the monotonic relationship between the random numbers used to generate inter-arrival and service times and the output statistic. It was reasonable to expect that a large random number would result in a large inter-arrival time and subsequently a shorter queue length as the result of a decreased arrival rate. Conversely, a small random number would result in a short inter-arrival time, or higher arrival rate, and subsequently a longer queue length.

However, the expected antithetic effects were entirely opposite for the generation of service times. That is, large random numbers would result in longer service times and longer queues. Whether or not the inter-arrival and service time effects would tend to cancel each other was not entirely clear at the beginning of the variance reduction experiments.

The results, as shown in Tables X, XI, and XII indicated that whether antithetics were applied to appointment arrivals, appointment times or both did not have a signficant effect on the variance reductions realized. All three antithetic methods were approximately equal in effect.

VI. CONCLUSIONS AND RECOMMENDATIONS

The following statements explain the point of view solely of the author and should not be construed as a reflection of the opinions of members of the FPC staff.

The statistical analysis showed that 8.7% of the doctor's total calendar time was unscheduled. More than half of this amount were time periods that had been withheld for semi-emergencies. Consequently, a 14% reduction of the time allocated to semi-emergencies could result in an increase of 4.4% (1469 minutes) in time available for scheduling routine appointments. This converts into 77 additional patient encounters per month, an increase of 7%. As it involves no changes in configuration or the total time the staff spends at the clinic, this relatively simple change is recommended.

A second change to increase patient capacity would be to lengthen the doctors' daily working hours from seven to eight. An additional 5040 minutes per month would be available for all activities. After subtracting 7.1% for no-shows and late cancellations, 8.7% for unscheduled time and 25.3% for OTHER activities, 2969 minutes remain for additional patient care each month. This translates into 156 more patient encounters per month, an increase of 14%. Since implementation of this change implies a change in clinic operating policy, it is presented for possible consideration only.

A third change, resulting in increased capacity, would be to reduce the percentage of time devoted to "OTHER" bona fide activities (25.3%). How this could be accomplished is beyond the purview of this study. A previous study of an outpatient clinic indicates that allocating only 15% to this activity would be feasible [Fetter and Thompson 1965]. A reduction of OTHER time from 25.3% to 15% translates into 148 more patients per month after time for no-shows and late cancellations and unscheduled time have been subtracted. This represents a 14% potential increase in the capacity of the doctors' patient population.

Results of the sensitivity analysis indicated that the most significant potential increase in patient capacity could be achieved by the addition of a physicians' assistant to the staff. As was shown in the section entitled PHYSICIANS' ASSISTANT, the desired PA utilization of one or two percentage points less than the overall system utilization could be realized by assigning 14% of each doctor's appointments to the PA. Therefore, without changing the percentages of time currently allocated as in Table III, a concomitant increase of 14% in the number of appointments would be possible. Coupled with the least effective change involving reduction of semi-emergency time, a potential 21% increase in patient appointments per month could be achieved. This alternative is highly recommended.

Considering the total amount of time and effort devoted to the variance reduction experiments, the results were

marginally effective at best. The control variate method should not be attempted unless a simple theoretical control model, with known expected values and a strong correlation to the original model, is available. Significant departures from the simple control model render the method inefficient in comparison with the method of antithetic variates. Changing a handful of computer cards was the only work involved in employing antithetics. The fact that approximately equal variance reductions were achieved whether antithetics were applied to arrivals only, appointment lengths only, or to both is an interesting result.

APPENDIX A

```
// EXEC GPSS,REGION=250K
//GO.DOUTPUT DD SYSOUT=A,SPACE=(CYL,(3,3))
//GO.SYSIN DD *
        REALLOCATE COM,84000
        REALLOCATE XAC,2000
        REALLOCATE VAR,65
*       FAMILY PRACTICE CLINIC      NORTH FORT ORD
*
        SIMULATE
 EXPON FUNCTION   RN1,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38/
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2/
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9997,8
 DIST1 FUNCTION   RN1,D4
.849,15/.990,30/.995,45/.999,60
 DIST2 FUNCTION   RN1,D4
.770,15/.944,30/.970,45/.999,60
 DIST3 FUNCTION   RN1,D4
.769,15/.949,30/.991,45/.999,60
 DIST4 FUNCTION   RN1,D4
.872,15/.979,30/.989,45/.999,60
     1 VARIABLE   X1-1
     2 FVARIABLE  FR1*210/100+1/2
     3 FVARIABLE  FR2*210/100+1/2
     4 FVARIABLE  FR3*210/100+1/2
     5 FVARIABLE  FR4*210/100+1/2
     6 FVARIABLE  FR9*210/100+1/2
     7 FVARIABLE  FR10*210/100+1/2
     8 FVARIABLE  FR11*210/100+1/2
     9 FVARIABLE  FR12*210/100+1/2
    10 FVARIABLE  (X2+X3+X4+X5+X54)/5460+1/2
    11 FVARIABLE  (X6+X7+X8+X9+X61)/5460+1/2
    12 VARIABLE   X2+X3+X4+X5+X6+X7+X8+X9
    13 VARIABLE   X49+X50+X51+X52+X54+X55+X61
    14 FVARIABLE  (546000-V12-V13)/5460+1/2
    15 FVARIABLE  X2/1092+1/2
    16 FVARIABLE  X3/1092+1/2
    17 FVARIABLE  X4/1092+1/2
    18 FVARIABLE  X5/1092+1/2
    19 FVARIABLE  (X2+X3+X4+X5+X54)*100/(546000-V13+X54+X61)+1/2
    20 FVARIABLE  X6/1092+1/2
    21 FVARIABLE  X7/1092+1/2
    22 FVARIABLE  X8/1092+1/2
    23 FVARIABLE  X9/1092+1/2
    24 FVARIABLE  (V12+X54+X61)*100/(546000-V13+X54+X61)+1/2
    25 FVARIABLE  X70/52+1/2
    26 FVARIABLE  X71/52+1/2
    27 FVARIABLE  X72/52+1/2
```

```
28 FVARIABLE  X73/52+1/2
29 FVARIABLE  X74/52+1/2
30 FVARIABLE  X2*100/(109200-X49)+1/2
31 FVARIABLE  X3*100/(109200-X50)+1/2
32 FVARIABLE  X4*100/(109200-X51)+1/2
33 FVARIABLE  X5*100/(109200-X52)+1/2
34 FVARIABLE  (X2+X6)*100/(109200-X49)+1/2
35 FVARIABLE  (X3+X7)*100/(109200-X50)+1/2
36 FVARIABLE  (X4+X8)*100/(109200-X51)+1/2
37 FVARIABLE  (X5+X9)*100/(109200-X52)+1/2
38 FVARIABLE  (X17+X18+X19+X20+X57)/52+1/2
39 VARIABLE   N$BEGIN
40 FVARIABLE  X47/52+1/2
41 FVARIABLE  FR13*210/100+1/2
42 FVARIABLE  FR14*210/100+1/2
43 FVARIABLE  FR15*210/100+1/2
44 FVARIABLE  FR16*210/100+1/2
45 FVARIABLE  (109200-X2-X6-X49)/1092+1/2
46 FVARIABLE  (109200-X3-X7-X50)/1092+1/2
47 FVARIABLE  (109200-X4-X8-X51)/1092+1/2
48 FVARIABLE  (109200-X5-X9-X52)/1092+1/2
49 FVARIABLE  X58+1
50 FVARIABLE  FR17*210/100+1/2
51 FVARIABLE  FR18*210/100+1/2
52 FVARIABLE  (109200-X54-X55-X61)/1092+1/2
53 FVARIABLE  X54*100/(109200-X55)+1/2
54 FVARIABLE  FR19*210/100+1/2
55 FVARIABLE  (X54+X61)*100/(109200-X55)+1/2
56 FVARIABLE  X54/1092+1/2
57 FVARIABLE  X61/1092+1/2
58 FVARIABLE  X75/52+1/2
59 FVARIABLE  X48/3
60 VARIABLE   Q1+Q2+Q3+Q4+Q17
61 FVARIABLE  X57/52+1/2
 1 MATRIX     H,53,6
 2 MATRIX     H,6,5
 1 STORAGE    1
 2 STORAGE    1
 3 STORAGE    1
 4 STORAGE    1
 5 STORAGE    1
 6 STORAGE    1
 7 STORAGE    1
 8 STORAGE    1
17 STORAGE    1
18 STORAGE    1
   INITIAL    X1,9/X2-X48,0
   INITIAL    X59,80
```

```
        INITIAL    LS1-LS8/LS17/LS19
        GENERATE   ,,,X59
        TRANSFER   ,BEGIN
        GENERATE   V1,FN$EXPON
        TRANSFER   .07,BEGIN,DEAD
DEAD    TERMINATE
        GENERATE   300,10
        LOGIC R    1
        GATE SE    1
        SEIZE      13
        ADVANCE    52
        LOGIC S    1
        RELEASE    13
        TERMINATE
        GENERATE   300,10
        LOGIC R    2
        GATE SE    2
        SEIZE      14
        ADVANCE    46
        LOGIC S    2
        RELEASE    14
        TERMINATE
        GENERATE   300,10
        LOGIC R    3
        GATE SE    3
        SEIZE      15
        ADVANCE    67
        LOGIC S    3
        RELEASE    15
        TERMINATE
        GENERATE   300,10
        LOGIC R    4
        GATE SE    4
        SEIZE      16
        ADVANCE    109
        LOGIC S    4
        RELEASE    16
        TERMINATE
        GENERATE   300,10
        LOGIC R    17
        GATE SE    18
        SEIZE      18
        ADVANCE    45
        LOGIC S    17
        RELEASE    18
        TERMINATE
BEGIN   TRANSFER   .545,SECND,FIRST
FIRST   TRANSFER   .545,TWO,ONE
```

```
SECND TRANSFER    .529,FOUR,THREE
  ONE TRANSFER    .01,UNO,FOUR
  UNO TRANSFER    .14,EINZ,PASS
 EINZ QUEUE       1
      ENTER       5
      GATE LS     1
      ENTER       1
      TRANSFER    .08,SHOW1,CNX1
SHOW1 GATE LS     5
      SEIZE       1
      DEPART      1
      ADVANCE     1,FN$DIST1
      RELEASE     1
      LEAVE       1
      LEAVE       5
      TERMINATE
 CNX1 SEIZE       9
      DEPART      1
      LOGIC R     5
      ADVANCE     17
      LOGIC S     5
      RELEASE     9
      LEAVE       1
      LEAVE       5
      TERMINATE
  TWO TRANSFER    .14,ZWEI,PASS
 ZWEI QUEUE       2
      ENTER       6
      GATE LS     2
      ENTER       2
      TRANSFER    .11,SHOW2,CNX2
SHOW2 GATE LS     6
      SEIZE       2
      DEPART      2
      ADVANCE     1,FN$DIST2
      RELEASE     2
      LEAVE       2
      LEAVE       6
      TERMINATE
 CNX2 SEIZE       10
      DEPART      2
      LOGIC R     6
      ADVANCE     20
      LOGIC S     6
      RELEASE     10
      LEAVE       2
      LEAVE       6
      TERMINATE
```

```
THREE TRANSFER   .14,DREI,PASS
 DREI QUEUE      3
      ENTER      7
      GATE LS    3
      ENTER      3
      TRANSFER   .085,SHOW3,CNX3
SHOW3 GATE LS    7
      SEIZE      3
      DEPART     3
      ADVANCE    1,FN$DIST3
      RELEASE    3
      LEAVE      3
      LEAVE      7
      TERMINATE
 CNX3 SEIZE      11
      DEPART     3
      LOGIC R    7
      ADVANCE    19
      LOGIC S    7
      RELEASE    11
      LEAVE      3
      LEAVE      7
      TERMINATE
 FOUR TRANSFER   .14,VIER,PASS
 VIER QUEUE      4
      ENTER      8
      GATE LS    4
      ENTER      4
      TRANSFER   .14,SHOW4,CNX4
SHOW4 GATE LS    8
      SEIZE      4
      DEPART     4
      ADVANCE    1,FN$DIST4
      RELEASE    4
      LEAVE      4
      LEAVE      8
      TERMINATE
 CNX4 SEIZE      12
      DEPART     4
      LOGIC R    8
      ADVANCE    17
      LOGIC S    8
      RELEASE    12
      LEAVE      4
      LEAVE      8
      TERMINATE
 PASS QUEUE      17
      ENTER      17
```

```
      GATE LS     17
      ENTER       18
      TRANSFER    .09,SHOWP,CNXP
SHOWP GATE LS     19
      SEIZE       17
      DEPART      17
      ADVANCE     30,5
      RELEASE     17
      LEAVE       18
      LEAVE       17
      TERMINATE
CNXP  SEIZE       19
      DEPART      17
      LOGIC R     19
      ADVANCE     30
      LOGIC S     19
      RELEASE     19
      LEAVE       18
      LEAVE       17
      TERMINATE
      GENERATE    2100
      SAVEVALUE   2+,V2
      SAVEVALUE   3+,V3
      SAVEVALUE   4+,V4
      SAVEVALUE   5+,V5
      SAVEVALUE   6+,V6
      SAVEVALUE   7+,V7
      SAVEVALUE   8+,V8
      SAVEVALUE   9+,V9
      SAVEVALUE   17+,FC1
      SAVEVALUE   18+,FC2
      SAVEVALUE   19+,FC3
      SAVEVALUE   20+,FC4
      SAVEVALUE   46,V38
      SAVEVALUE   47+,V39
      SAVEVALUE   48,V40
      SAVEVALUE   49+,V41
      SAVEVALUE   50+,V42
      SAVEVALUE   51+,V43
      SAVEVALUE   52+,V44
      SAVEVALUE   54+,V50
      SAVEVALUE   55+,V51
      SAVEVALUE   57+,FC17
      SAVEVALUE   58,V49
      SAVEVALUE   59,V59
      SAVEVALUE   60,V61
      SAVEVALUE   61+,V54
      SAVEVALUE   70+,Q1
```

```
SAVEVALUE  71+,Q2
SAVEVALUE  72+,Q3
SAVEVALUE  73+,Q4
SAVEVALUE  74+,Q17
SAVEVALUE  75+,V60
MSAVEVALUE 1,53,1,V25,H
MSAVEVALUE 1,53,2,V26,H
MSAVEVALUE 1,53,3,V27,H
MSAVEVALUE 1,53,4,V28,H
MSAVEVALUE 1,53,5,V29,H
MSAVEVALUE 1,53,6,V58,H
MSAVEVALUE 1,X58,1,Q1,H
MSAVEVALUE 1,X58,2,Q2,H
MSAVEVALUE 1,X58,3,Q3,H
MSAVEVALUE 1,X58,4,Q4,H
MSAVEVALUE 1,X58,5,Q17,H
MSAVEVALUE 1,X58,6,V60,H
MSAVEVALUE 2,1,1,V15,H
MSAVEVALUE 2,2,1,V16,H
MSAVEVALUE 2,3,1,V17,H
MSAVEVALUE 2,4,1,V18,H
MSAVEVALUE 2,5,1,V56,H
MSAVEVALUE 2,6,1,V10,H
MSAVEVALUE 2,1,2,V20,H
MSAVEVALUE 2,2,2,V21,H
MSAVEVALUE 2,3,2,V22,H
MSAVEVALUE 2,4,2,V23,H
MSAVEVALUE 2,5,2,V57,H
MSAVEVALUE 2,6,2,V11,H
MSAVEVALUE 2,1,3,V45,H
MSAVEVALUE 2,2,3,V46,H
MSAVEVALUE 2,3,3,V47,H
MSAVEVALUE 2,4,3,V48,H
MSAVEVALUE 2,5,3,V52,H
MSAVEVALUE 2,6,3,V14,H
MSAVEVALUE 2,1,4,V30,H
MSAVEVALUE 2,2,4,V31,H
MSAVEVALUE 2,3,4,V32,H
MSAVEVALUE 2,4,4,V33,H
MSAVEVALUE 2,5,4,V53,H
MSAVEVALUE 2,6,4,V19,H
MSAVEVALUE 2,1,5,V34,H
MSAVEVALUE 2,2,5,V35,H
MSAVEVALUE 2,3,5,V36,H
MSAVEVALUE 2,4,5,V37,H
MSAVEVALUE 2,5,5,V55,H
MSAVEVALUE 2,6,5,V24,H
TERMINATE  1
```

	COL. 1	2	3	4	5	6
ROW 1	0	0	1	0	14	15
2	10	3	5	11	19	48
3	6	11	0	25	0	42
4	33	27	11	25	13	109
5	48	28	7	25	7	115
6	60	36	12	26	3	137
7	46	1	0	14	1	62
8	57	5	13	30	15	120
9	69	7	24	26	36	162
10	71	17	23	23	56	190
11	91	35	24	17	64	231
12	108	37	34	11	45	235
13	123	37	39	16	36	251
14	111	16	32	1	25	185
15	100	1	7	3	15	126
16	122	7	28	15	14	186
17	126	17	23	20	20	206
18	110	7	19	8	14	158
19	117	13	19	11	10	170
20	132	28	25	6	2	193
21	120	5	18	0	16	159
22	97	3	4	0	0	104
23	94	3	1	3	0	101
24	103	1	0	0	0	104
25	91	5	2	17	0	115
26	81	2	0	5	5	93
27	69	7	0	0	0	76
28	74	22	0	2	12	110
29	77	15	1	3	2	98
30	64	2	2	2	2	72
31	57	1	0	2	2	62
32	55	3	5	11	0	74
33	34	0	0	7	3	44
34	21	6	5	1	10	43
35	2	3	6	3	10	24
36	0	0	0	1	0	1
37	2	1	1	2	0	6
38	3	0	1	0	0	4
39	2	3	1	11	2	19
40	11	12	0	8	0	31
41	3	3	3	4	0	13
42	5	8	14	3	7	37
43	0	0	5	7	0	12
44	17	8	4	19	6	54
45	24	16	29	27	14	110
46	37	4	30	33	33	137
47	3	0	20	0	10	33
48	7	0	6	4	0	17
49	6	8	11	1	7	33
50	24	11	45	4	15	99
51	10	2	26	4	1	43
52	3	0	6	10	1	20
53	53	9	11	10	11	94

MATRIX HALFWORD SAVEVALUE 2

	COL. 1	2	3	4	5
ROW 1	74	6	3	90	97
2	67	9	9	79	89
3	66	6	5	85	93
4	52	7	5	81	92
5	69	7	10	81	89
6	66	7	6	83	92

LEGEND

1st Matrix

Columns	
1	Doctor 1 queue
2	Doctor 2 queue
3	Doctor 3 queue
4	Doctor 4 queue
5	PA queue
6	Total System queue
Rows	
1-52	Queue length measured at end of each week

2nd Matrix

Columns	
1	% of total time caring for patients
2	% of total time for no-shows, late cancellations
3	% of total time not scheduled
4	Doctor utilization (%)
5	Calendar utilization (%)
Rows	
1	Doctor 1
2	Doctor 2
3	Doctor 3
4	Doctor 4
5	PA
6	Total System

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